Solutions to JEE MAIN-1|JEE 2024

PHYSICS

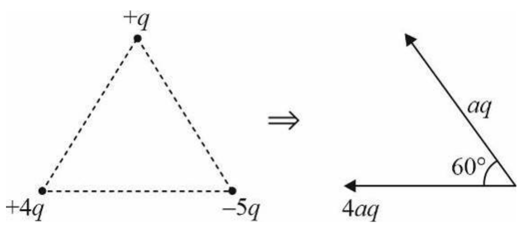
SECTION-1

1.(C) $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2};\ F' = \frac{1}{4\pi\varepsilon_0 k}\frac{q^2}{2a^2}$

Therefore, the net force on charge at B is $F_B = F\sqrt{2} + F'$

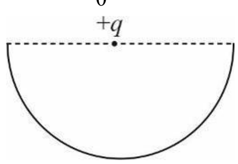
$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\sqrt{2} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2} = \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

2.(D)



Net dipole moment $= \sqrt{(4aq)^2 + (aq)^2 + 2(aq)(4aq)\cos 60^\circ} \quad = \sqrt{21}aq$

3.(C) $\phi = \frac{q}{2\in_0}$



4.(B) Since net force on negative charge is always directed towards fixed positive charge, the torque on negative charge about positive charge is zero. Therefore angular momentum of negative charge about fixed positive charge is conserved.

5.(D) By conservation of energy, we use $U_i + K_i = K_f + U_f$

$$0 + \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r} + 0 \quad \Rightarrow \quad r \propto \frac{1}{v^2}$$

If v is doubled, the minimum distance r will become one fourth.

6.(B) We consider a spherical shell of thickness dx and radius x. The volume of this spherical shell is $4\pi x^2 dx$. The charge enclosed within shell is given as

$$dq = \left[\frac{Qx}{\pi R^4}\right][4\pi x^2 dx] = \frac{4Q}{R^4}x^3 dx$$

The charge enclosed in the sphere of radius r_1 is given as

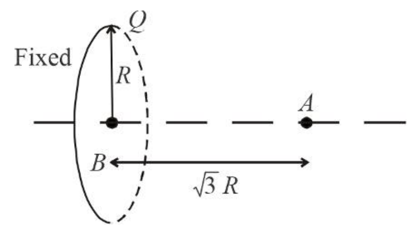
$$q = \frac{4Q}{R^4}\int_0^{r_1} x^3 dx = \frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1} = \frac{Q}{R^4}r_1^4$$

The electric field at point P inside the sphere at a distance r_1 from the centre of the sphere is given as

$$E = \frac{1}{4\pi\in_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2} \quad \Rightarrow \quad E = \frac{1}{4\pi\in_0}\frac{Q}{R^4}r_1^2$$

7.(D) As potential due to uniformly charged ring at its axis (at x distance) is $V = \frac{kQ}{\sqrt{R^2+x^2}}$

So, potential at point A due to ring ; $V_1 = \frac{kQ}{\sqrt{R^2+3R^2}} = \frac{kQ}{2R}$



So potential energy of charge $-q$ at point A

$P.E_1 = \frac{-kQq}{2R}$ and potential at point B, $V_2 = \frac{kQ}{R}$

So, potential energy of charge $-q$ at point B : $P.E_2 = \frac{-kQq}{R}$

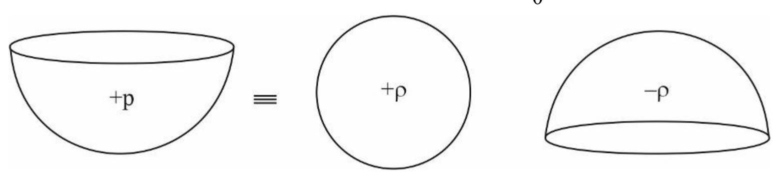
Now by energy conservation : $P.E_1 + K.E_1 = P.E_2 + K.E_2$

$\frac{-kQq}{2R} + 0 = \frac{-kQq}{R} + \frac{1}{2}mv^2 \quad \Rightarrow \quad V^2 = \frac{kQq}{mR}$

So velocity of charge $-q$ at point B $\quad V = \sqrt{\frac{kQq}{mR}}$

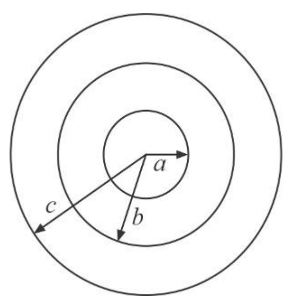
8.(B) Apply principle of superposition

Electric field due to a uniformly charged sphere $= \frac{\rho R}{12\varepsilon_0}$



$E_{\text{Resultant}} = \frac{\rho R}{12\varepsilon_0} - E$

9.(D) $\sigma(4\pi a^2 + 4\pi b^2 + 4\pi c^2) = Q$



$4\pi\sigma(a^2+b^2+c^2) = Q$

$V = \frac{1}{4\pi\varepsilon_0} \times \frac{\sigma 4\pi a^2}{a} + \frac{\sigma 4\pi b}{4\pi\varepsilon_0} + \frac{\sigma 4\pi c}{4\pi\varepsilon_0}$

$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}(a+b+c) = \frac{Q}{4\pi\varepsilon_0}\frac{(a+b+c)}{(a^2+b^2+c^2)}$

10.(C) Conceptual

11.(D) Remember that the field at the centre of a circular arc of radius R and subtending angle θ at its centre, with total charge Q uniformly distributed over it, is

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\left(\frac{\sin(\theta/2)}{\theta/2}\right)$$

Therefore, the field at the centre of the given semi-circular ring, $E_1 = \frac{\lambda(\pi R)}{4\pi\varepsilon_0 R^2}\left(\frac{1}{\pi/2}\right) = \frac{\lambda}{2\pi\varepsilon_0 R}$

We know that this field will point perpendicular to the diameter of the semi-circular ring, away from the ring if the charge is positive and towards the ring if the charge is negative.

So, the field due to the two halves will point in the same direction and add up. Therefore, the net field at the centre, $E = 2E_1 = \frac{\lambda}{\pi\varepsilon_0 R}$

12.(C) $E_{axis} = \frac{1}{4\pi \in_0} \cdot \frac{qx}{(R^2 + x^2)^{3/2}}$

For maximum E, $x = \pm \frac{R}{\sqrt{2}}$

$E_{\max} = \frac{1}{4\pi \in_0} \cdot \frac{2q}{3\sqrt{3}R^2}$

13.(B)

14.(A) For equipotential surface

$$-\int \vec{E} \cdot \vec{dl} = -\int Edl\cos\frac{\pi}{2} = 0 \quad \Rightarrow \quad \Delta V = 0 \quad \Rightarrow \quad W_{ext} = q\Delta V = 0$$

15.(C) $U = \frac{1}{2}\varepsilon_0 E^2$ (volume) (here electric field is uniform)

$$U = \frac{1}{2}\varepsilon_0 \left(\frac{\sigma}{2\varepsilon_0}\right)^2 \left(\frac{4}{3}\pi r^3\right)$$

$$U = \frac{4\pi r^3 \sigma^2}{8\varepsilon_0 \times 3}; \quad U = \frac{\pi \sigma^2 r^3}{6\varepsilon_0}$$

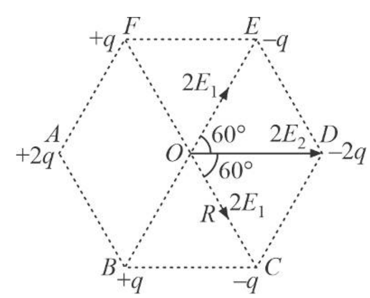
16.(C) **(i)** $V_A = \frac{Kq}{R_2}$

(ii) $E_A = O$ (point is inside metallic conductor)

(iii) $E_B = \frac{Kq}{CB^2}\widehat{CB}$

(iv) $F_Q = \frac{KQq}{CB^2}\widehat{CB}$

17.(A) If E_1 is the electric field at O due to $-q$ at E directed from O to E and E_2 is the electric field at O due to $+2q$ at A directed from O to D then the net electric field at O is given as



$$E = 2E_1\cos 60° + 2E_1\cos 60° + 2E_2$$

$$\Rightarrow \quad E = E_1 + E_1 + 2E_2 = 2E_1 + 2E_2$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{L^2} + \frac{1}{4\pi\varepsilon_0}\frac{4q}{L^2}$$

$$\Rightarrow \quad E = \frac{6}{4\pi\varepsilon_0}\frac{q}{L^2} \text{ along } OD$$

If electric potential at point O is V, then it is given by sum of all the potentials at O due to all individual charges given as

$$V = \frac{-q}{4\pi\varepsilon_0 L} - \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{q}{4\pi\varepsilon_0 L} + \frac{2q}{4\pi\varepsilon_0 L} - \frac{2q}{4\pi\varepsilon_0 L} = 0$$

For line PR all the charges are symmetrically located at same distance from O thus potential at all points of the PR must be same. Thus option (A), (B) and (C) are correct.

18.(B)

19.(B) Distribution of charge on different surfaces of the plates has been shown.
Take a point P on the leftmost plate. The electric field at P is

$$E = \frac{Q-q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{(q-2Q)}{2A\varepsilon_0} = 0$$

20.(D) The charge on the removed element is given as

$$dq = \frac{Q(dL)}{2\pi a}$$

The electric field due to the charge on the element at the centre, is given as

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{dq}{a^2} = \frac{1}{4\pi\epsilon_0}\left[\frac{QdL}{2\pi a^3}\right]$$

We know that electric field at the centre of a uniformly charged circular loop is zero. If E_2 be the electric field of the remaining wire loop, then we use

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{0} \quad \Rightarrow \quad \vec{E}_2 = -\vec{E}_1 \quad \Rightarrow \quad E_2 = E_1$$

$$E_2 = \frac{QdL}{8\pi^2 \epsilon_0 a^3}$$

SECTION-2

1.(500) Flux will be

$$\phi = E.A. \qquad = (6\hat{i} + 2\hat{j} + 5\hat{k}) \cdot (100\hat{k}) = 500$$

2.(80) $64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ (r : radius of small drop, R : radius of big drop)

$$R = 4r$$

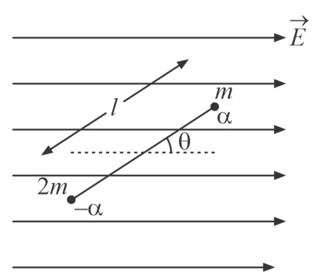
Now, $\frac{Kq}{r} = 5$

For big drop

$$V = \frac{K(nq)}{R} \qquad = \frac{K(64q)}{4r}$$

$$= \frac{16Kq}{r} \qquad = 16 \times 5 = 80 \text{ volts}$$

3.(6) $PEQ = I\alpha$; $qlEQ = \left\{2m\left(\frac{l}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2\right\}\alpha$



$$\Rightarrow \quad qlEQ = \left(\frac{2ml^2}{9} + \frac{4ml^2}{9}\right)\alpha \quad \Rightarrow \quad qlEQ = \frac{6ml^2}{9}\alpha$$

$$\Rightarrow \quad qEQ = \frac{2ml}{3}\alpha \quad \Rightarrow \quad \alpha = \left(\frac{3qE}{2ml}\right)Q$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3qE}{2ml}}$$

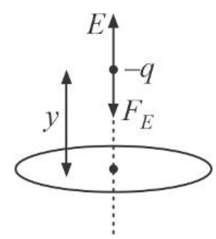
4.(1) Field inside the cylinder, $E_{in}(r) = \frac{\rho r}{2\varepsilon_0}$

Therefore, $E\left(\frac{R}{2}\right) = \frac{\rho R}{4\varepsilon_0}$

Field outside the cylinder, $E_{out}(r) = \frac{\rho R^2}{2\varepsilon_0 r}$

Therefore, $E(2R) = \frac{\rho R}{4\varepsilon_0}$

5.(4) When the negative charge is shifted at a distance y from the centre of the ring along its axis then force acting on the point charge due to the ring.



$F_E = qE$ (towards centre) $= q\left[\frac{KQy}{(a^2+y^2)^{3/2}}\right]$

If $a >> y$ then $a^2 + y^2 \simeq a^2$

$\therefore \quad F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qqy}{a^3}$ (towards centre)

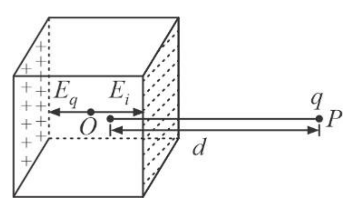
Since, restoring force $F_E \propto y,$ therefore motion of charge the particle will be S.H.M.

Time period of SHM

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 m a^3}{Q \times q}\right]^{1/2}$$

6.(1) Here $\vec{E}_i = \vec{E}$ electric field due to induced charges and E_q = electric field due to charge q

We know that net electric field in a conducting cavity is equal to zero

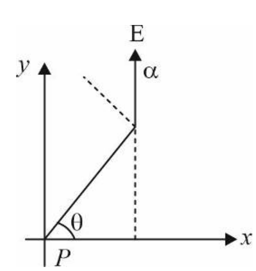


i.e. $\vec{E} = \vec{0}$ at the centre of the cube

$\Rightarrow \quad \vec{E}_i + \vec{E}_q = \vec{0}$

$\Rightarrow \quad \vec{E}_i = -\vec{E}_q \quad \Rightarrow \quad \vec{E}_i = -\frac{kq}{d^2}\overline{PO}$

7.(16) $\vec{E}_{\text{axis}} = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$ (along $\vec{P}$)



$E_{\text{bisector}} = \frac{1}{4\pi\varepsilon_0}\frac{P}{(2r)^3}$

(opposite to $\vec{P}$)

So, $E_2 = -\frac{\vec{E}_1}{16}$

8.(0) Potential is a scalar quantity. The charge density function suggests that net charge is zero. Net charge on the ring

$$Q = \int_0^{2\pi} \lambda R d\theta \;= \lambda_0 R \int_0^{2\pi} \cos(\theta/2) d\theta = 0$$

$$V = -\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right) = 0$$

9.(32) Total charge $Q = 80 + 40 = 120\mu C$. By using the formula $Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$.

New charge on sphere *A* is $Q_A = Q\left[\frac{r_A}{r_A + r_B}\right] = 120\left[\frac{4}{4+6}\right] = 48\mu C$.

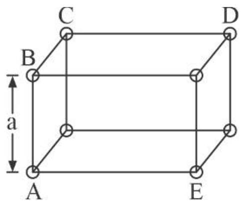
Initially it was $80\mu C$ i.e., $32\mu C$ charge flows from *A* to *B*.

10.(2) $E = \frac{kQ_1R}{\left(\sqrt{R^2+R^2}\right)^3} + \frac{kQ_2R}{\left(\sqrt{R^2+3R^2}\right)^3} = 0 \qquad \Rightarrow \qquad \frac{Q_1}{Q_2} = -\frac{1}{2\sqrt{2}}$

CHEMISTRY

SECTION-1

1.(A)



If concerned particle is A then

$AB = a$ (nearest)

$AC = \sqrt{2}a$ (next-nearest)

$AD = \sqrt{3}a$ (next-next-nearest)

2.(D) Orthorhombic contains:
Simple, body centre, face centre and end centre bravais lattices.

3.(C) In fcc structure, corner atoms do not touch each other (atoms 1 and 2), but every face centre atom touches corners. Moreover, every face centre atom touches every other face centre atom provided it is not the opposite face centre atom in an fcc unit cell.

(i) Atoms 3 and 4 are touching each other where centre-to-centre distance $= a/\sqrt{2}$

(ii) Atom 1 and 2 are not touching each other

(iii) Atoms 2 and 4 are touching each other where centre-to-centre distance $= a/\sqrt{2}$

4.(A) **(i)** Edge length = AB = AD = BC = CD = a

(ii) $AC = \sqrt{2}a$

(iii) AG (body diagonal) $= \sqrt{3}a$ **(iv)** Therefore $AA' = AG/2 = \frac{\sqrt{3}}{2}a$

5.(C) For an ideal solution

$\Delta H_{mixing} = 0, \Delta V_{mixing} = 0, \Delta S_{mixing} > 0$ and it should obey Raoult's law.

6.(B)

Gas	Temperature	K_H(K bar)
He	293	144.97
N_2	293	76.48
O_2	293	34.86

Higher the value of Henry's Law constant, the lower is the solubility of the gas in the liquid at constant partial pressure of gas.

7.(D) $\Delta T_f = 2.55°C = K_f \cdot m \quad \therefore \quad m = \frac{2.55}{K_f}$

$\Delta T_b = K_b m$

$\Rightarrow \quad \Delta T_b = 2.55 \times \frac{K_b}{K_f} = 2.55 \times \frac{0.52}{1.86} °C = 0.7°C$

$\Rightarrow \quad T_b = 100 + 0.7 = 100.7°C$

8.(B)

$$A_n \rightleftharpoons nA$$

1

$1-\alpha \qquad n\alpha$

$i = 1-\alpha+\alpha/n \qquad\qquad i = 1-\alpha+\alpha n$

$$\Rightarrow \quad \alpha = \frac{i-1}{n-1}$$

9.(C)

10.(B)

11.(C)

12.(A) Theoretical

13.(C)

14.(D) **(i) – (c)**

$H_2O + H_2SO_4$ = Azeotropic mixture (at a particular composition) shows negative deivation.

(ii) – (d)

$C_6H_6 + H_2O \rightarrow$ Immiscible mixture

(iii) – (b)

$C_2H_5Br + C_2H_5I$ = Ideal solution

(iv) – (a)

$C_2H_5OH + H_2O$ = Azeotropic mixture (at a particular composition) shows positive deviation.

15.(B) Theoretical **p**

16.(B) Theoretical

17.(C) Theoretical

18.(A)

19.(C) Reason is correct explanation for assertion.

20.(C) Reason is correct explanation for assertion.

SECTION-2

1.(2) $\Delta T_b = iK_b m$

$$0.27 = i \times 0.54 \times \frac{12.2}{122} \times \frac{1000}{100}$$

or $\quad i = 0.5$

Therefore, benzene associated as dimer, i.e., 2

2.(14) Let $\quad \pi_1 = 200\text{ mm}, T_1 = 283$

$\pi_2 = 150.3\text{ mm}, T_2 = 298$

Now, $\quad \pi = \frac{n}{V}RT\,p$

At T_1, $200 = \frac{n}{V_1} \times R \times 283$

At T_2, $150.3 = \frac{n}{V_2} \times R \times 298$

3.(4) Total colligative properties are four.

4.(7) For ZnS structure, Z = 4

Number of $B^{\Theta} = 4/\text{unit}$ cell (corner + face centre)

Number of $A^{\oplus} = 4/\text{unit}$ cell (in alternate TVs)

Number of B^{Θ} ion removed = 4 (Two from each face centre) $\times\frac{1}{2}$ (per face centre share) = 2

Number of B^{Θ} ions left = 4 – 2 = 2/unit cell

Number of Z^{2-} ions entering in place of $B^{\Theta} = 1$

[To maintain electrical neutrality, $2B^{\Theta} = 1Z^{2-}$]

Formula $= A_4B_2Z_1$ $\quad\therefore\quad$ $x + y + c = 4 + 2 + 1 = 7$

5.(4) Na_2O has fcc structure $\quad\therefore\quad Z = 4/\text{unit cell}$

$\therefore$ Formula $= 4Na_2O = Na_8O_4$ $\quad\therefore\quad$ Coordination number of $Na^{\oplus} = 4$

Note: CN of cation = Number of anions
CN of anion = Number of cations

Antifluorite-type structures have (4 : 8) coordination number and $Na^{\oplus}$ ions are in all TVs.

6.(48) Truncated octahedron has 8 hexagonal faces and 6 square faces.

7.(34) P.F. of dimond $= \dfrac{n_{eff.}(V_{atom})}{V_{cube}} = \dfrac{8\left[\frac{4}{3}\pi R^3\right]}{a^3} = \dfrac{8\left[\frac{4}{3}\pi R^3\right]}{\left(\frac{8R}{\sqrt{3}}\right)^3} = 0.34$

Hence, packing efficiency = 0.34 × 100 = 34

8.(19) $x_A = \frac{1}{2}, x_B = \frac{1}{2};\qquad P_T = P_A^\circ\frac{1}{2} + P_B^\circ \times \frac{1}{2}$

(Given $P_A^\circ = 20$)

$90 = 45 \times 2 = P_A^\circ + P_B^\circ \qquad \dots \text{(i)}$

$P_B^\circ = 90 - 20 = 70$

$22.5 = P_A^\circ x_A + P_B^\circ(1 - x_A) = 20x_A + 70(1 - x_A);\qquad 22.5 = 20x_A + 70 - 70x_A = 70 - 50x_A$

$x_A = \frac{47.5}{50} = \frac{19}{20};\qquad x_B = \frac{1}{20};\qquad \frac{x_A}{x_B} = \frac{19/20}{1/20} = 19$

9.(4) $\pi = CRT = \frac{nRT}{V}$

Given, $w = 40\text{ g};\qquad M = 246$

$T = 27°C = 300K;\qquad V = 1\text{ L}$

Substituting all the values, we get

$\pi = \frac{40}{246} \times 0.082 \times 300 = 4\text{ atm}$

10.(12)

MATHEMATICS

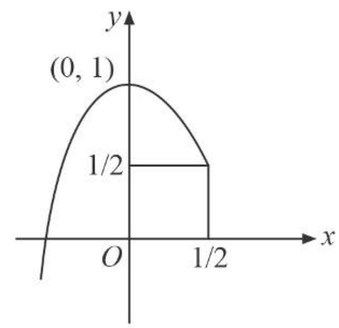
SECTION-1

1.(D) $f(x)=\frac{1}{2}(2\sqrt{1-2x}+2x)$

$=-\frac{1}{2}(-2\sqrt{1-2x}-2x)$ $\quad=-\frac{1}{2}((\sqrt{1-2x}-1)^2-2)$

$y=1-\frac{1}{2}[(\sqrt{1-2x}-1)^2]$

$y_{\max}=1,$ when $x=0$



$x = 0$ is the inflection point

Alternative:

$f'(x)=1-\frac{1}{\sqrt{1-2x}}$; $f'(x)=0$

$\Rightarrow \quad 1-2x=1 \quad \Rightarrow \quad x=0$

Also, $\quad f(-\infty)=\rightarrow-\infty$

OR

Put $\quad \sqrt{1-2x}=t$; $\quad 1-2x=t^2$

$x=\frac{1-t^2}{2}$; $\quad f(t)=t+\frac{1-t^2}{2} \quad =\frac{2t+1-t^2}{2}=\frac{2-(t-1)^2}{2}$

2.(A) $F(x)=\begin{cases}-x^2 & \text{if} \quad x\ge 1\\ -x^2 & \text{if} \quad x\le -1\\ 1 & \text{if} \quad x\in(-1,1)-\{0\}\\ 2 & \text{if} \quad x=0\end{cases} \quad \Rightarrow \quad F(x)$ is even

3.(C) If $f(x)<0$ then $|f(x)|=-f(x)$

Hence, $-f(x)>-f(x)$ is not possible

If $f(x)>0$ then $|f(x)|=f(x)$

Hence, $f(x)>-f(x)$

$\Rightarrow \quad 2f(x)>0 \quad \Rightarrow \quad f(x)>0$

$\Rightarrow \quad x^2-4x+3>0 \quad \Rightarrow \quad x\in(-\infty,\ 1)\cup(3,\infty)$

4.(D) (i), (iii) and (iv) are clearly odd, for (ii) we use

$\left(\sqrt{x^6+1}+x^3\right)\left(\sqrt{x^6+1}-x^3\right)=1$

5.(B) $\log_2(x^2+3x)\le 2$

$\Rightarrow \quad x^2+3x\le 4$

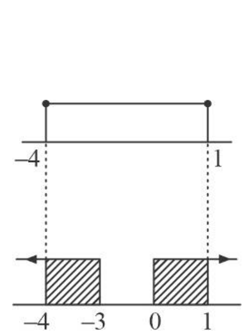
$\Rightarrow \quad x^2+4x-x-4\le 0$

$\Rightarrow \quad (x+4)(x-1)\le 0$

$\Rightarrow \quad -4\le x\le 1$

Also, $\quad x^2+3x>0$

$\Rightarrow \quad x(x+3)>0 \quad \Rightarrow \quad x>0 \quad$ or $\quad x<-3$



6.(C) $y = (7\cos\theta + 24\sin\theta) \times (7\sin\theta - 24\cos\theta)$

$r\cos\phi = 7, r\sin\phi = 24$

$r^2 = 625, \tan\phi = \frac{24}{7}$

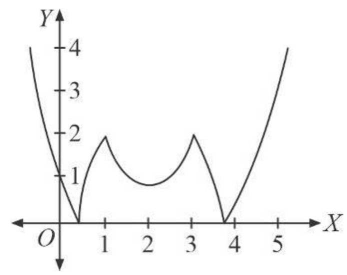
$y = r\cos(\theta - \phi) \cdot r\sin(\theta - \phi)$

$= \frac{r^2}{2} \cdot 2\sin(\theta - \phi)\cos(\theta - \phi)$

$= \frac{r^2}{2} \cdot (\sin 2(\theta - \phi))$

$y_{\max} = \frac{25^2}{2} = \frac{625}{2}$

7.(C) See graph $y = f(x) = ||x^2 - 4x + 3| - 2|$, $y = m$ is a horizontal line with intersection points, from which the x-values have different signs, only if $m > 2$.
Also, verify remaining options.



8.(D) LCM of $\left(\frac{2\pi}{2\pi}, \frac{2\pi}{\pi/3}, \frac{2\pi}{\pi/5}\right) = 30$

9.(B) One root lies in the interval $\left(-\frac{\pi}{2}, 0\right)$

10.(B) $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} \Rightarrow \sqrt{3} = \tan 3 \times \frac{\pi}{9}$

$= \frac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1 - 3\tan^2\frac{\pi}{9}}$

$\Rightarrow \quad \left[\sqrt{3}\left(1 - 3\tan^2\frac{\pi}{9}\right)\right]^2 = \left(3\tan\frac{\pi}{9} - 3\tan^3\frac{\pi}{9}\right)^2$

$\Rightarrow \quad \tan^6\frac{\pi}{9} - 33\tan^4\frac{\pi}{9} + 27\tan^2\frac{\pi}{9} = 3 = \tan^2\frac{\pi}{3}$

11.(C) Given $f(x + ay, x - ay) = axy$

Let $x + ay = u$ and $x - ay = v$

Then, $x = \frac{u + v}{2}$ and $y = \frac{u - v}{2a}$

Substituting the value of x and y in (i), we obtain

$f(u, v) = \frac{u^2 - v^2}{4} \quad \Rightarrow \quad f(x, y) = \frac{x^2 - y^2}{4}$

12.(B) We have, $f(x) = 4x^x + 2^x + 1$

Let $y = 2^{2x} + 2^x + 1 \quad \Rightarrow \quad 2^{2x} + 2^x + 1 - y = 0$

$\Rightarrow \quad 2^x = \dfrac{-1 + \sqrt{1 - 4(1-y)}}{2} \qquad \therefore \; x = \log_2\left(\dfrac{\sqrt{4y-3} - 1}{2}\right)$

Which defined, when $4y - 3 \geq 0$

$\Rightarrow \quad y \geq \dfrac{3}{4}$ ....(i)

And $\dfrac{\sqrt{4y-3} - 1}{2} \Rightarrow \sqrt{4y-3} > 1$

$\Rightarrow \quad y > 1$ ....(ii)

From equations, (i) and (ii), we get:

$\therefore$ Range of $f(x) = (1, \infty)$

13.(A) $fog(x) = x^3 - \dfrac{1}{x^3}$

$$f(g(x)) = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

Let $x - \dfrac{1}{x} = t, \; f(t) = t^3 + 3t$

Thus, $f(x) = x^3 + 3x$

14.(A) $\dfrac{\sqrt{4 - x^2}}{1 - x} > 0, 4 - x^2 > 0 \quad \Rightarrow \quad x \in (-2, 1)$ and range is [–1, 1]

15.(A) $\cos\theta + \sin\theta = \dfrac{1}{5}, \; 0 < \theta < \pi$; $\dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} + \dfrac{2\tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} = \dfrac{1}{5}$

$\Rightarrow \quad 5 - 5\tan^2\dfrac{\theta}{2} + 10\tan\dfrac{\theta}{2} = 1 + \tan^2\dfrac{\theta}{2} \quad \Rightarrow \quad 6\tan^2\dfrac{\theta}{2} - 10\tan\dfrac{\theta}{2} - 4 = 0$

$\Rightarrow \quad 6\tan^2\dfrac{\theta}{2} - 12\tan\dfrac{\theta}{2} + 2\tan\dfrac{\theta}{2} - 4 = 0 \quad \Rightarrow \quad \tan\dfrac{\theta}{2} = 2, \; \tan\dfrac{\theta}{2} = -\dfrac{1}{3}; \qquad \theta \in (0, \pi)$

So, $\dfrac{\theta}{2} \in \left(0, \dfrac{\pi}{2}\right)$. Hence $\tan\dfrac{\theta}{2} = -\dfrac{1}{3}$ is discarded $\tan\dfrac{\theta}{2} = 2$

$$\Rightarrow \quad \tan\theta = \frac{2\tan\left(\frac{\theta}{2}\right)}{1 - \tan^2\left(\frac{\theta}{2}\right)} = \frac{4}{1-4} = -\frac{4}{3}$$

16.(D) (A) $\text{sgn}(e^{-x}) = 1$

(B) LCM (π, π) is π

(C) Minimum $(|x|, \sin x) = \sin x$

(D) Simplify to get $-\left\{x + \frac{1}{2}\right\} - \left\{x - \frac{1}{2}\right\} - 2\{-x\}$

17.(D) $N^r = \sin 2\theta + \sin 4\theta - \sin 6\theta$, where $\theta = \frac{\pi}{7}$

$= 2\sin 3\theta \cos\theta - 2\sin 3\theta \cos\theta$

$= 2\sin 3\theta (\cos\theta - \cos 3\theta)$

$= 2\sin 3\theta (2\sin 2\theta \sin\theta)$

$= 4\sin\theta \sin 2\theta \sin 3\theta$

$N^r = \underbrace{4\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\sin\frac{5\pi}{7}}_{D^r} \Rightarrow \frac{N^r}{D^r} = 4 \quad \left(\text{As} \quad \sin\frac{2\pi}{7} = \sin\frac{5\pi}{7}\right)$

18.(B) $\cos x = \sqrt{1 - \sin 2x} = |\sin x - \cos x|$

(i) $\sin x \le \cos x \Rightarrow \cos x = \cos x - \sin x \Rightarrow \sin x = 0$

where, $x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$\therefore \quad \sin x = 0 \Rightarrow x = 0 \Rightarrow x = 2\pi$, neglecting $x = \pi$

(ii) $\sin x > \cos x \Rightarrow \tan x = 2$

where $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right) \quad \therefore \quad \tan x = 2 \Rightarrow x = \tan^{-1}(2)$

Thus, the given equation has two solution

19.(A) If $f(x) = 2$

$\Rightarrow \quad \sin x = 1, \ \sin x\sqrt{3} = 1 \quad \Rightarrow \quad x = (4n+1)\frac{\pi}{2}$ and $x\sqrt{3} = (4m+1)\frac{\pi}{2}$

$\Rightarrow \quad \sqrt{3} = \frac{4m+1}{4n+1}$ which is a contradiction as LHS is irrational whereas RHS is rational

$\therefore \quad f(x)$ cannot attain value2

$\therefore$ 2 cannot be the maximum value of $f(x) \quad \Rightarrow \quad \sin x = \sin x\sqrt{3} = -1$

Which is again impossible by same reason

So, (B) is true,

Now period of $f(x) = LCM\left(2\pi, \frac{2\pi}{\sqrt{3}}\right)$ which does not exist as multiples of 2π are $\pm 2\pi, \pm 4\pi, \ldots$

whereas multiplies of $\frac{2\pi}{\sqrt{3}}$ are $\pm\frac{2\pi}{\sqrt{3}}, \pm\frac{4\pi}{\sqrt{3}}, \pm\frac{6\pi}{\sqrt{3}}$

Therefore, (C) is false

Now, $f(0) = 0 \quad \therefore \quad f(x) > 0 \ \forall \ x \in R$ is false, i.e., (D) is false

20.(D) Given, $af(x)+bf\left(\frac{1}{x}\right)=x-1,\ x\neq 0,\ a\neq b$ ....(i)

$\Rightarrow \quad af\left(\frac{1}{x}\right)+bf(x)=\frac{1}{x}-1$ ....(ii)

$a\cdot(i)-b\cdot(ii)$

$\Rightarrow \quad (a^2-b^2)f(x)=a(x-1)-b\left(\frac{1}{x}-1\right)$

$\Rightarrow \quad (a^2-b^2)f(2)=a+\frac{b}{2}=\frac{2a+b}{2}$

$\Rightarrow \quad f(2)=\frac{2a+b}{2(a^2+b^2)}$

SECTION-2

1.(0) $2P_6-3P_4+1=2(1-3\sin^2 x\cos^2 x)-3\left[(\sin^2 x+\cos^2 x)-2\sin^2 x\cos^2 x\right]+1$

$=2-6\sin^2 x\cos^2 x-3(1-2\sin^2 x\cos^2 x)+1=0$

2.(2) Discuss between integers $\quad\therefore\quad x=0$ and $x=2$

3.(9) $|\tan \pi y|+(\sin \pi x)^2=0$ … (i)

and $\quad x^2+y^2\leq 2$ … (ii)

Equation (i) is possible only if

$\tan \pi y=0$

$\Rightarrow \quad \pi y=m\pi$

$\Rightarrow \quad y=m(m\in I)$

and $\quad \sin^2 \pi x=0$

$\Rightarrow \quad \pi x=n\pi$

$\Rightarrow \quad x=n(n\in I)$

Equation (2), $x^2+y^2\leq 2$

Hence, $x, y\in[-\sqrt{2},\sqrt{2}]$

$\therefore$ Possible values of x : {–1, 0, 1}

and possible values of y : {–1, 0, 1}

$\therefore$ Total number of ordered pairs (x, y) is 3 × 3 = 9

4.(4) Given $f(x)=\begin{cases} x & , -2\leq x\leq -1 \\ x^2+2x & , -1<x\leq 0 \\ 2x-x^2 & , 0<x\leq 1 \\ 2-x & , 1<x\leq 2 \end{cases}$

$\therefore$ From above graph, range of $f(x)=[-2,1]$

Hence, number of integers in the range of $f(x)$ are 4

i.e., $\quad -2, -1, 0, 1$

5.(1) $\therefore\ g(x) = 1 + x - [x] = 1 + \{x\} \geq 1\ \forall\ x \in R$

By definition of $f(x) = 1$, for $x > 0$

$f(g(x) \geq 1) = 1$

6.(4) $\cos 3x = 4\cos^3 x - 3\cos x$

$\cos x \neq 0$

7(2) We have, $f(x) = (x+a) - [x+b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots$

$$+ \sin(2n-1)\pi x + \cos 2n\pi x$$

To period of $x + a - [x+b] + b - b \Rightarrow x + b - [x+b] + a - b \Rightarrow \{x+b\} + a - b$

Hence, we see that its period is 1

Now, period of $\sin \pi x$ is $\dfrac{2\pi}{\pi} = 2$

Period of $\cos 2\pi x$ is $\dfrac{2\pi}{2\pi} = 1$

Similarly, period of $\cos 2n\pi x = \dfrac{2\pi}{2n\pi} = \dfrac{1}{n}$

$\therefore$ Period of $f(x)$ is LCM of all above period, which is 2

8.(5) We know that

$$[x] + [-x] = \begin{cases} -1, & x \notin I \\ 0, & x \in I \end{cases} \quad \text{and} \quad \text{RHS is either } +1 \text{ or } -1$$

$\therefore$ above equation valid only when

$$\frac{\log_3(x-2)}{|\log_3(x-2)|} = -1 \quad \Rightarrow \quad \log_3(x-2) = \text{ negative}$$

$\Rightarrow \quad 0 < x - 2 < 1 \quad \Rightarrow \quad 2 < x < 3$

$\Rightarrow \quad x \in (2, 3) \quad \Rightarrow \quad a = 2, b = 3$

9.(28) $f(x) = \dfrac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$

$$= \underbrace{ax^7 + bx^5 + cx^3 + dx + \frac{1}{x}}_{\text{odd function}} + 15$$

Now, $f(x) + f(-x) = 30$ or $f(-5) = 30 - f(5) = 28$

10.(54) $f(3x + 2) + f(3x + 29) = 0$ …(i)

Replacing x by $x + 9$, we get

$$f(3(x+9)+2) + f(3(x+9)+29 = 0$$

or $f(3x+29) + f(3x+56) = 0$ …(ii)

from (i) and (ii), we get

$f(3x + 2) = f(3x + 56)$

or $f(3x + 2) = f(3(x + 18) + 12)$

Therefore, $f(x)$ is periodic with periodic 54